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Comparative Study Between Lognormal and Weibull Distributions in Modeling Commercial Concentrator III–V Triple-Junction Solar Cells Lifetimes

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Abstract- The paper attracts the attention that the accelerated lifetime of commercial concentrator lattice match triple-junction GaInP/GaInAs/Ge cells is better fitted by lognormal than Weibull distribution that has been adopted by most of the researchers in the field. A fair number of statistical tests are used to analyze a real-time dataset from accelerated life testing (ALT) that significantly favors the lognormal distribution. For comparison purposes, the Arrhenius-Weibull and lognormal stress relationships are used to predict the lifetime model under nominal conditions. They provide comparable estimates to the nominal meantime to failure (MTTF) and activation energy of the cells; yet, the two models possess different behaviors, especially at their tails and peaks. Moreover, an intensive Monte Carlo simulation is conducted to examine the distribution robustness towards censoring. The results again affirm that the censored samples of Lognormal are more efficient than those of Weibull in estimating the distribution parameters.

Keywords Accelerated life test; Arrhenius life-stress relationship; Concentration photovoltaic systems; Efficiency of censored samples; Goodness-of-fit tests; Lognormal distribution; Solar cells; Weibull distribution.

1. Introduction

The growth of energy demands due to technological and demographical expansion has led to more pollution resulting in global warming associated with many environmental and natural disasters worldwide. Concerns about the changing environment and fossil fuel depletion have prompted much controversy and scrutiny of more green energy. In particular, solar power riveted vast attention in the last several years at the forefront in reducing greenhouse gas emissions. Driven by the world's energy demands, photovoltaic cells are on course to accelerate renewable technologies. Under the scope of this growth, the concentrated photovoltaic (CPV) technology continues to achieve an unparalleled efficiency of 47.1%, far beyond the traditional flat plate technology [1,2]. The minimizing thermalization and absorption losses may push efficiency even higher and provide a potential pathway for the production of solar electricity at a reduced cost [3,4]. Nonetheless, reliability remains one of the challenges hindering further proliferation of this technology. Long-term stability and reliability are the bottlenecks for establishing confidence in designing engineering disciplines. In the context of performance analysis, reliability is also a key objective for cost competitiveness and commercialization of renewable energy devices and systems, delivering economical and quality power. Achieving such objectives necessitates a proper analysis and modeling of the system's lifetime.

CPV is a relatively new technology, leading to a scarcity of long-term historical knowledge about its lifetime characteristics and performance degradation. Since standard lifetime tests can span several decades in data collection, it is impractical to wait that long to assess reliability. Hence, accelerated lifetime tests (ALTs) are engineered to obtain life data by mimicking the field failure modes within an acceptable time frame. ALTs are implemented in the laboratories by applying more severe stresses than actual use conditions by exposing the CPVs to different accelerating stresses, such as temperature, current, and voltage. The accelerated data derived from ALTs is then used to extrapolate and characterize the product lifetime model and performance under operating conditions. The lifetime model is a function of time that describes the underlying probabilistic distribution of failure events. It reveals essential information about the product, such as the projected warranty, returns cost, and the MTTF. Improper fit of the distribution model or wrong estimation of its parameters can fail to reflect the life data analysis accurately, hence drawing false conclusions. As such, the performance of a CPV needs an accurate prediction, which subsequently allows energy industries to carry out strategic plans related to the maintenance or replacement of CPV systems, given the efficiency of solar cells has been obsolete. Henceforth, serious efforts must be devoted to building an appropriate lifetime distribution model for CPV cells.

This work re-evaluates the accelerated lifetime testing data of CPV under different thermal stresses for 45 commercial concentrator lattice match triple-junction GaInP/GaInAs/Ge cells [5]. As a general trend, Weibull is the classical distribution used in literature to model solar cells and electronics lifetime [6,7,8,9,10,11,12,13]. Many advocate the selection of Weibull solely to its popularity in reliability assessment, commercial software availability, and flexibility. In addition, few papers used other distributions deprived of scientific proof [14,15,16,17,18]. The absence of a solid statistical basis raises a question about the suitability of other parametric distributions.

To our knowledge, this work has not been done previously. Unlike the aforementioned works, we provide a valuable insight into the lifetime modeling of the concentrator III–V triple-junction solar cells in order to motivate and aid in the development of reliable CPV modules with a well-defined warranty. The main contribution of this work is a novel and holistic statistical analysis that discriminates between lognormal and Weibull distributions in fitting to CPV experimental data through a comprehensive comparative framework, supported by standard tests and indicators. Leveraging on our obtained results, we provide efficient entropy-based estimators of the MTTF and warranty times of photovoltaic modules, demonstrating that lognormal is a better lifetime model for CPV cells. The rest of the paper is structured as follows: In Sec. 2, the experimental ALT is clarified, considering the temperature as the primary accelerating variable and determinant of cell degradation. For the convenience of readers, Sec. 3 recalls the elementary functions of the two distribution models. Sec. 4 conducts several goodness-of-fit tests to confirm that both distributions successfully fit the experimental data showing that the lognormal distribution is significantly favorable. Sec. 5 utilizes the Arrhenius stress relationship in estimating the lifetime models under practical conditions for both probability distributions. Based on the derived models at operating conditions, an intensive simulation is performed in Sec. 6 to validate that the censored samples are highly efficient estimators when the lifetime model is assumed to be lognormally distributed. Lastly, Sec. 7 summarizes and concludes the paper.

2. Experiment

The experimental data is adopted from [5]. In [5], a sample of 45 commercial concentrators lattice-matched GaInP/GaInAs/Ge cells was equally segregated and exposed to three temperature levels; T1: 164°C (437K), T2: 126°C (399K), and T3: 119°C (392K). The time a cell was able to endure that temperature was recorded, reflecting the cell lifetime. The operation was replicated by injecting current in the darkness, which is equivalent to the photogenerated current by CPV subjected to the actual field irradiance of 820X. The failure was identified by checking the dark I-V (current-voltage) curve, an effective approach for examining the current-voltage characteristics of the solar cells in dark conditions. At the lowest temperature intensity, T3, ALT took a long time to induce failures, so the experiment was terminated after the failure of the 9th cell. This segment of the CPV cells is considered censored as some are still operating. Thus, their exact lifetimes are unknown, whereas the two higher temperature levels, T1 and T2, resulted in the failure of all the CPVs. Once all the observed failure times, t, were recorded, statistical treatment was employed to deduce the CPV population lifetime model. Fig. 1 and Fig. 2 depict the probability plots of the observed failure times, which graphically show the compliance of the data to the two-lifetime models, Weibull and Lognormal. Nevertheless, probability plots are deemed subjective [19]; a comprehensive analysis is accomplished when combining graphical and analytical techniques. The latter is the subject of the following sections.

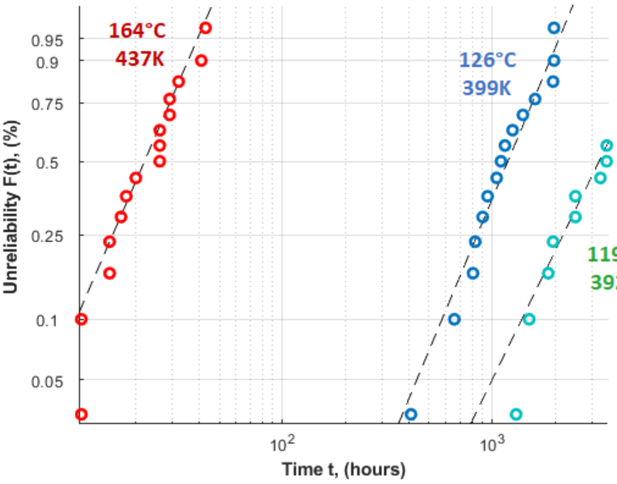

Fig. 1. Unreliability of the ALT data as a function of time at the three stress levels for Weibull.

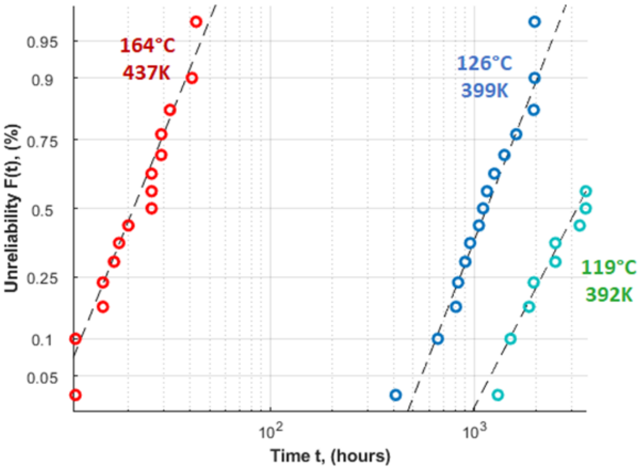

Fig. 2. Unreliability of the ALT data as a function of time at the three stress levels for lognormal.

3. Weibull and lognormal distributions

For the convenience of readers, this section recalls the probability density function (PDF), cumulative distribution function (CDF), mean and mode, and the maximum likelihood estimating equations (MLEs) from complete sample $\{X_1, X_2, \dots, X_n\}$ and type I right censored samples $\{X_1, X_2, \dots, X_r\}, r \le n$ for both distributions [20].

3.1 *Weibull distribution*

Weibull distribution, denoted in this paper by Weibull (α, β) where $\beta \in (0, \infty)$ is the scale parameter, and $\alpha \in (0, \infty)$ is the shape parameter, has the following statistical terms:

PDF: $$f(x) = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} exp\left[-\left(\frac{x}{\beta}\right)^{\alpha}\right], x \ge 0. \quad (1)$$

CDF: $$F(x) = 1 - exp\left[-\left(\frac{x}{\beta}\right)^{\alpha}\right], x \ge 0 \quad (2)$$

Mean: $$= \beta\Gamma\left(1 + \frac{1}{\alpha}\right), \quad (3)$$

where $\Gamma(.)$ is the gamma function.

Mode: $$\vartheta = \beta\left(\frac{\alpha-1}{\alpha}\right)^{\frac{1}{\alpha}}, \text{when } \alpha > 1. \quad (4)$$

MLEs from complete samples:

$$\frac{\sum_{i=1}^{n} x_i^{\hat{\alpha}_{com}} \log x_i}{\sum_{i=1}^{n} x_i^{\hat{\alpha}_{com}}} - \frac{1}{\hat{\alpha}_{com}} = \frac{1}{n}\sum_{i=1}^{n} \log x_i, \quad (5a)$$

$$\hat{\beta}_{com} = \left(\frac{1}{n}\sum_{i=1}^{n} x_i^{\hat{\alpha}_{com}}\right)^{1/\hat{\alpha}_{com}}. \quad (5b)$$

MLEs from censored samples:

$$r = (n-r)\left(t/\hat{\beta}_{cen}\right)^{\hat{\alpha}_{cen}} \log\left(t/\hat{\beta}_{cen}\right) - \sum_{i=1}^{r}\left[1 - \left(x_i/\hat{\beta}_{cen}\right)^{\hat{\alpha}_{cen}}\right] \log\left(x_i/\hat{\beta}_{cen}\right)^{\hat{\alpha}_{cen}}, \quad (6a)$$

$$r = (n-r)\left(t/\hat{\beta}_{cen}\right)^{\hat{\alpha}_{cen}} + \sum_{i=1}^{r}\left(x_i/\hat{\beta}_{cen}\right)^{\hat{\alpha}_{cen}}. \quad (6b)$$

3.1 *Lognormal distribution*

The lognormal distribution is denoted by Lognormal (μ, σ) where $\mu \in (-\infty, \infty)$ is the scale parameter, and $\sigma \in (0, \infty)$ is the shape parameter, has the following statistical terms:

PDF: $$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} \quad exp\left[\frac{(\log x - \mu)^2]}{2\sigma^2}\right], \ x \ge 0. \quad (7)$$

CDF: $$F(x) = \frac{1}{2} + \frac{1}{2} erf\left[\frac{\log x - \mu}{\sqrt{2}\sigma}\right], \ x \ge 0. \quad (8)$$

Mean: $$\theta = exp\left(\mu + \frac{\sigma^2}{2}\right). \quad (9)$$

Mode: $$\vartheta = exp(\mu - \sigma^2). \quad . \quad (10)$$

MLEs from complete samples:

$$\hat{\mu}_{com} = \frac{1}{n}\sum_{i=1}^{n} \log x_i, \quad (11a)$$

$$\hat{\sigma}_{com} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} (\log x_i - \hat{\mu}_{com})^2}. \quad (11b)$$

MLEs from censored samples:

$$\hat{\mu}_{cen} = \frac{1}{r}\sum_{i=1}^{r} \log x_i + \frac{n-r}{r}\frac{\phi(z_{cen})}{1-\varphi(z_{cen})}\hat{\sigma}_{cen}, \quad (12a)$$

$$\hat{\sigma}_{cen} = \sqrt{\frac{1}{r}(\log t - \hat{\mu}_{cen})\sum_{i=1}^{r}(\hat{\mu}_{cen} - \log x_i) + \frac{1}{r}\sum_{i=1}^{r}(\log x_i - \hat{\mu}_{cen})^2}, \quad (12b)$$

where $z_{cen} = \frac{\log t - \hat{\mu}_{cen}}{\hat{\sigma}_{cen}}$, ϕ and φ are the PDF and CDF of the standard normal distribution, respectively.

The scale parameter determines the steepness of the failure density distribution curve and exhibits the length of object life. On the other hand, the shape parameter directly affects the geometric shape of the failure density distribution curve, which reflects the failure mechanism of objects.

4. Goodness of model

Three of the most powerful goodness-of-fit tests are used in evaluating how well the lognormal and Weibull distributions fit the three datasets obtained from ALT. The hypothesis tests considered in this work are Anderson-Darling (AD) [22,23], Cramer-von Mises (CvM) [24,25], and Jarque-Bera (JB) [26]. The first two tests are commonly used in the test of fit of a family of distributions, including Weibull and lognormal distributions, whereas JB is only used to confirm the lognormality of the data. As the distributions' parameters are estimated from the experimental data, adjusting the test statistics and critical values of the tests must be taken into consideration. Therefore, an intensive simulation approach is chosen to implement these tests in order to avoid obtaining any wrong results. CvM and AD test statistics belong to the class of quadratic statistics [21] that have the following general form

$$Q = n\int_{-\infty}^{\infty}\{F_n(x) - F(x)\}^2\psi(x)dF(x), \qquad (13)$$

where n is the observed sample size, $F_n(x)$ is the CDF of the sample, $F(x)$ is the CDF of the candidate theoretical distribution, and $\psi(x)$ is a suitable weight function. When $\psi(x) = 1$, the statistics Q becomes the CvM statistics, and when $\psi(x) = \{F(x) - F^2(x)\}^{-1}$, it becomes the AD statistics, which are respectively given in equations Eq. (14) and Eq. (15). CvM places more weight to the center of the distribution, whereas AD amplifies the effect of the tail. Therefore, together would give a strong reason to accept or reject a candidate distribution. The test statistics of AD and CvM are given by:

$$AD = -n - \frac{\sum_{i=1}^{n}(2i-1)\left(Log(z_i) + Log(1 - z_{n+1-i})\right)}{n}. \qquad (14)$$

$$CVM = \frac{1}{12n} + \sum_{i=1}^{n}\left(\frac{2i-1}{2n} - z_i\right)^2. \qquad (15)$$

where $z_i = F(x_i)$, $i = 1,2,\dots,n$, are the ranks of the observed values x_i under $F(x)$, and that of JB is as follows:

$$JB = n\left(\frac{s^2}{6} + \frac{(k-3)^2}{24}\right), \qquad (16)$$

where s and k are skewness and kurtosis parameters of the log-transformed data, respectively. The JB is commonly applied as a complementary test in checking the validity of lognormality besides the aforementioned tests. This test is not suitable for assessing Weibull distribution.

The tests are implemented using Monte Carlo Simulation by generating a huge number of samples of 20000 to achieve precise estimates for the critical values and p-values. For the censored third stress (T3) data, modified tests are used as in [24] and [29] for the Cramer-von Mises, Anderson-Darling, and the Kolmogorov-Smirnov. Also, the estimates of the moments used in the Jarque-Bera test are obtained from censored data [30]. The results of the tests are shown in Tables 1-3 for the three data sets, respectively, in agreement with that of [31], which primarily investigated the problem. The distribution that scores a higher p-value is favorable as it reflects a closer fit to the data. A 5% level of significance is assumed. Tables 1-3 use the following abbreviations, TS: Test statistic, CV: Critical Value, PV: p-value, W: Weibull, and L: Lognormal.

Table 1. Goodness-of-fit tests summary for the data set T_1

	Weibull			Lognormal			
Test	TS	CV	PV	TS	CV	PV	Best Fit
AD	0.318	0.736	0.570	0.327	0.732	0.543	W
CvM	0.046	0.120	0.579	0.053	0.126	0.597	L
JB		NA		0.662	3.214	0.589	

Table 2. Goodness-of-fit tests summary for the data set T_2

	Weibull			Lognormal			
Test	TS	CV	PV	TS	CV	PV	Best Fit
AD	0.358	0.729	0.466	0.247	0.733	0.769	L
CvM	0.049	0.121	0.541	0.024	0.124	0.557	L
JB		NA		0.545	3.384	0.682	

Table 3. Goodness-of-fit tests summary for the data set T_3

	Weibull			Lognormal			
Test	TS	CV	PV	TS	CV	PV	Best Fit
AD	0.148	0.410	0.528	0.136	0.379	0.554	L
CvM	0.025	0.071	0.448	0.025	0.068	0.458	L
JB		NA		3.456	4.587	0.334	

From Tables 1-3, one can see that both distributions fit ALT datasets well. However, all indicate that the lognormal is the best fitting model as it achieves a higher p-value for AD and CvM in almost all cases. Furthermore, JB strongly recommends lognormal. JB is deemed a strong evidence for lognormality when the logarithm of the sample is proved to be normally distributed [27,28]. Therefore, according to the results above, it can be concluded that, at least, the given CPV data is better fitted by lognormal than Weibull.

5. Predicting the lifetime at operating conditions

Since the ALT samples of the stresses T_1 and T_2 are complete, the MLEs for both distributions are calculated from Eq. (5) and Eq. (11). On the other hand, the data of stress T_3 is censored, hence its MLEs are calculated from Eq. (6) and Eq. (12). The termination time for stress T_3 is set as t=3600 h, corresponding to the 9th cell failure at 3515 h. The results are outlined in Table 4.

Table 4. MLEs vs. stress levels

Stress level	MLEs: Weibull	MLEs: Lognormal
T_1 =164°C/437 K	$\beta_1 = 27.0$, $\alpha_1 = 2.709$	$\mu_1 = 3.0920$, $\sigma_1 = 0.415$
T_2=126 °C/399 K	$\beta_2 = 1351.8$, $\alpha_2 = 2.764$	$\mu_2 = 7.0062$, $\sigma_2 = 0.424$
T_3 =119 °C/392 K	$\beta_3 = 3745.8$, $\alpha_3 = 2.602$	$\mu_3 = 8.0673$, $\sigma_3 = 0.518$

It can be easily seen from Table 4 that the life scale increases as the stress decreases, where the life scale is represented by β for Weibull and μ for lognormal. On the other hand, the failure mechanism exhibits the same style for the three stresses as the shape parameters, α or σ, are very similar and free of the stress T. Usually, the Arrhenius-type model is vastly used to calculate thermal acceleration in the life-stress relationship [32], which is governed by:

$$l(T) = e^{a+b/T} , \tag{17}$$

where l: is any life measure such as the characteristic, median, or mean lives, T: denotes the stress, which is the temperature in Kelvin, and a, b: are the model acceleration parameters to be obtained.

The linear form of the Arrhenius model can be obtained by taking the logarithms of both sides of Eq. (17), which reduces to:

$$\log l = a + b\frac{1}{T} \tag{18}$$

The common life measure for Weibull is assumed to be ($l = \beta$), which is the 63rd percentile of the distribution, whereas it is the median ($l = e^{\mu}$) for lognormal distribution [19]. Hence, the Arrhenius-Weibull and Arrhenius–Lognormal models can respectively be written as:

$$\text{Weibull: } \log\beta = a + b\ \frac{1}{T} \tag{19}$$

$$\text{Lognormal: } \mu = a + b\ \frac{1}{T} \tag{20}$$

The key assumptions that validate using Arrhenius stress relationship models Eq. (19) and Eq. (20) are:

(A1) The three datasets must acceptably fit the Weibull or lognormal lifetime model, which has been confirmed in Sec. 4.

(A2) The failure mechanism is free of the stresses T_1, T_2, T_3 and the use stress, where the applied stress only changes the scale of the lifetime. This implies that the shape parameters of the lifetime distributions, for all thermal stress levels and use stress, must be the same:

$$\text{Weibull: } \alpha = \alpha_1 = \alpha_2 = \alpha_3 \tag{21}$$

$$\text{Lognormal: } \sigma = \sigma_1 = \sigma_2 = \sigma_3 \tag{22}$$

This is a strong assumption that is practically not possible to hold precisely. Therefore, the weighted average of the shape parameters at all stresses would be a reasonable estimation of the shape parameters at normal operating conditions, given as the following:

$$\text{Weibull: } = \frac{\sum_{i=1}^{3} n_i \alpha_i}{\sum_{i=1}^{3} n_i} = 2.7055 , \tag{23}$$

$$\text{Lognormal: } \sigma = \frac{\sum_{i=1}^{3} n_i \sigma_i}{\sum_{i=1}^{3} n_i} = 0.4642, \tag{24}$$

where $n_1 = n_2 = 15$ and $n_3 = 9$ as clarified in Sec. 2.

(A3) The relationship between $\log\beta_i$ and $1/T_i$, and, μ_i and $1/T_i$ should be sufficiently linear. This is proven using the linear correlation coefficient (ρ) between $\log\beta_i$ and $1/T_i$, and, between μ_i and $1/T_i$. But since $logT_i$ is common, it is enough to calculate ρ between $\log\beta_i$ and μ_i, which is in this case, ρ=0.99, as can be easily checked from Table 5.

Table 5. The relationship between $\mathbf{1/T_i}$ and $\mathbf{log\beta_i}$ and $\boldsymbol{\mu_i}$

	$T_1 = 437$	$T_2 = 399$	$T_3 = 392$
$1/T_i$	0.0022883	0.002506	0.0025510
$\log\beta_i$	3.2958	7.2092	8.2284
μ_i	3.0920	7.0062	8.0559

The accelerating parameters a and b in Eq. (19) and Eq. (20) are obtained using the least square method. For the Weibull model ($a = -39.1065$, $b = 18522.4435$), whereas for lognormal ($a = -39.5016$, $b = 18604.9327$), and the two equations take the following forms:

$$\text{Weibull: } \log\beta = -39.1065 + 18522.4435\ \frac{1}{T} \tag{25}$$

$$\text{Lognormal: } \mu = -39.5016 + 18604.9327\ \frac{1}{T} \tag{26}$$

The scale parameters corresponding to each distribution under operating conditions are then determined by substituting $T = 353\ K$ in Eq. (25) and Eq. (26):

$$\text{Weibull: } \beta = 637309.9 \tag{27}$$

$$\text{Lognormal: } \mu = 13.2036 \tag{28}$$

The accelerating parameter b in the Arrhenius model Eq. (17) hides a critical parameter E

$$b = \frac{E}{k}, \tag{29}$$

where E (eV) is the activation energy, which is the minimum energy needed to induce the failure mechanism, and $k = 8.6173303 \times 10^{-5}\text{eVK}^{-1}$ is the Boltzmann's constant. The activation energy can be calculated for both models by substituting the values of b from Eq. (25) and Eq. (26) in Eq. (29). It can be checked that the estimates of E from Weibull and lognormal are E=1.59614 eV and E=1.60325 eV, respectively, which are in excellent agreement with E =1.59 eV derived in [5]. Indeed, activation energies of solar cells obtained in other works [6,33,34,35,36] range between 0.5 to 1.75 eV, depending on the fabrication material and processing. Interestingly, similar activation energies are reported for several optoelectronic devices [37,38,39,40,41].

Table 6 illustrates the figures of particular interest to investors and manufacturers under operating conditions in view of Weibull and lognormal distributions. The figures are the estimated parameters, the mean time to failure θ, the mode time, defined as the most frequent failure time ϑ, and 5% and 10% warranty times expressed as W_5 and W_{10}, respectively. The times are provided in hours (h) and years (y) after assuming five daily working hours under 80°C $(353\,K)$ temperature.

Table 6. The estimations of some key figures under operating conditions obtained from the two lifetime distributions

Figure	Weibull	Lognormal
Shape	α =2.7055	σ =0.4642
Scale	β = 637309.9	μ = 13.2036
θ	566788 h 310 y	604007 h 330 y
ϑ	537379 h 294 y	447096 h 240 y
W_5	212602 h 116 y	252725 h 138 y
W_{10}	277406 h 152 y	299151 h 164 y

6. Performance of censored data

To improve the design of ALTs, larger samples and less stress levels are usually considered. Larger samples are expected to be more representative of the population under consideration, whereas less stress would activate failures more naturally as they would be in reality and better imitate the use environment. However, these two factors would definitely lead to a significant expansion in the duration of the ALTs, where some objects are anticipated to have a long lifetime, making it difficult to be observed given the cost and time constraints. Hence, to save time and reduce cost, it might be convenient to terminate the ALT at a pre-assumed time t and be content with observing the lifetime of objects that fail before the time t. In the literature, this technique is known as type I right censoring. Mathematically speaking, when a complete ordered sample $X_{com} = \{X_1, X_2, \ldots, X_n\}$ of lifetimes is right-censored at time t, the resultant right-censored sample can be described as $X_{cen}(t) = \{X_1, X_2, \ldots, X_r\}$, which comprises of the lifetimes that are less than or equal to t, that is, $X_{(i)} \le t$, for all i=1,2, …, r, where $r \le n$ is the size of the censored sample. Censoring schemes would definitely save time and cost, yet, they may as well cause a significant loss of information that would harm the estimation as a post-process when the termination happens too early. On the other hand, late termination would waste time, and hence the whole process becomes useless. Hence, the efficiency of censored samples must be carefully examined and assessed. In addition, the efficiency of censored samples is significantly affected by the assumed statistical model of the experimental data, where it is expected that censored samples of the best fitting model are most efficient.

In this context, an additional advantage of the lognormal distribution is added as its censored samples will be proved to be more efficient in estimating the MTTF than those of Weibull distribution. Considering the following criteria, the performance of censored samples can be evaluated:

1) The relative error in estimating the mean time to failure.

To conduct a sensible comparison, it might not be convenient to compare the parameters of Weibull to those of lognormal. A more logical approach compares the predicted MTTF resulting from Weibull and lognormal since they refer to the same information and have the same physical meaning [42] in both models. Let the estimated values of the MTTF be denoted by θ_{com} and $\theta_{cen}(t)$ from a complete sample and its censored sample at time t, respectively, the simple relative error in estimating the MTTF is

$$\epsilon_t = |\theta_{com} - \theta_{cen}(t)|/\theta_{com}. \tag{30}$$

It can be noticed that the complete sample is mandatory to be fully observed in order to compute the relative error, which is not the case in practical applications as in the case of the third stress T_3. Therefore, a more practical criterion is needed for that purpose.

2) The entropy-based efficiency of the censored sample.

The use of entropy to measure the amount of information in the truncated distributions [43,44,45] and censored samples [46,47,48,49,50] has been widely employed in the literature. This paper applies sup-entropy [47] through its corresponding efficiency of the censored sample, defined by:

$$\tau_t = \frac{E[\log(f(X_{cen}(t))/\delta_{cen})]}{E[\log(f(X_{com})/\delta_{com})]}, \tag{31}$$

where

$$f(x_{com}) = n! \prod_{i=1}^{n} f(x_{(i)}), \tag{32}$$

$$f(x_{cen}(t)) = d[1 - F(t)]^{n-r} \prod_{i=1}^{r} f(x_{(i)}), \tag{33}$$

where d denotes the ordering constant, $f(x_{com})$ and $f(x_{cen}(t))$ are the likelihood probability density functions of the complete and censored samples [52] and, δ_{com} and δ_{cen} are their supremum values, respectively.

Table 7. Performance comparison of type I censored samples under operating conditions

	Weibull					Lognormal				
t	r/n	τ_t	θ_{com}	$\theta_{cen}(t)$	ϵ_t	r/n	τ_t	θ_{com}	$\theta_{cen}(t)$	ϵ_t
550000	0.490	0.407	566776	678406	0.199	0.513	0.519	598451	620929	0.114
650000	0.651	0.437	567006	665959	0.174	0.659	0.666	597990	609588	0.067
750000	0.788	0.480	567541	643884	0.133	0.768	0.772	598924	607466	0.048
850000	0.888	0.585	567302	618608	0.088	0.847	0.849	597853	603199	0.032
950000	0.947	0.722	567851	598649	0.053	0.900	0.900	596781	601223	0.022
1050000	0.979	0.844	566985	581788	0.025	0.934	0.933	596632	599910	0.016
1150000	0.993	0.924	567375	573418	0.010	0.956	0.955	598162	600278	0.011

Since the efficiency function depends on the distributions' parameters, they are endeavored to be estimated from the censored sample. The estimates will be acceptable only if the data is not excessively censored, which would happen when the termination time exceeds the mode of the distribution [42].

The efficiency Eq. (31) is an increasing function of t and bounded by zero and one for all probability distributions [30], which justifies its applicability in judging the performance of censored samples with respect to complete samples.

An intensive Monte Carlo simulation is adopted in this section to investigate the performance of censored samples under field conditions from Weibull and lognormal distributions relying on the aforementioned criteria Eq. (30) and Eq. (31). The complete sample size is set to be n=15, matching the experimental sample's size at each stress.

To avoid excessive censoring, the initial termination time selected was 550000 hours as it surpasses the modes of the two distributions. Then, t increments gradually with a step size of 100000 hours until it reaches its maximum at 1150000 hours. The simulation process is described through the following steps:

(1) A completely random sample of size n=15 is generated from both distributions having parameters reported in Table 6.

(2) The censored samples are formed by selecting the values that are less than or equal to t in step (1).

(3) The censored sample fraction r/n is calculated, where r denotes the size of the censored sample determined in step (2).

(4) The MLEs of the parameters from the complete samples in step (1) are computed using Eq. (5) and Eq. (11) of Weibull and lognormal distributions, respectively.

(5) The MLEs of the parameters from the censored samples in step (2) are computed using Eq. (6) and Eq. (12) of Weibull and lognormal distributions, respectively.

(6) The MLEs of the MTTF are calculated by substituting the parameters' MLEs computed from the complete samples in step (4) in Eq. (3) and Eq. (9) of Weibull and lognormal distributions, respectively.

(7) The MLEs of the MTTF are computed by substituting the parameters' MLEs computed from the censored samples in step (5) in Eq. (3) and Eq. (9) of Weibull and lognormal distributions, respectively.

(8) The relative error in estimating the MTTF ϵ_t is computed using Eq. (30) for both distributions.

(9) The efficiency of censored sample τ_t is calculated by substituting the parameters' MLEs of the censored samples in step (5) in Eq. (31) for both distributions.

(10) Steps (1-9) are repeated M times, M=5,000 and the average values of the figures r/n, ϵ_t and τ_t are calculated and reported in Table 7.

According to Table 7, the efficiency of the censored sample τ_t increases to one as the time *t* increases, while the relative error in estimating the MTTF ϵ_t decreases to zero with *t*. In addition, the estimated values of the MTTF from the complete samples θ_{com} are close to that of complete experimental sample θ reported in Table 6, and the estimated MTTFs of the simulated censored sample $\theta_{cen}(t)$ converge to θ_{com} as t increases. Based on the values of τ_t and ϵ_t, lognormal significantly exhibits higher suitability towards censoring than Weibull, despite the similar sample size percentages r/n of both distributions. Weibull would start to deliver comparable estimates to lognormal only at very late termination times at which almost all objects are observed, and the censoring becomes useless.

It is clear from Table 7 that the censored samples of the lognormal distribution are more efficient in estimating the MTTF; this can be easily seen by observing that

$$\tau_t(Lognormal) > \tau_t(Weibull) \qquad (30)$$

and

$$\epsilon_t(Lognormal) < \epsilon_t(Weibull), \qquad (31)$$

for almost all termination times t, though the censored sample size of the lognormal distribution is in general comparable or less than that of the Weibull distribution.

7. Conclusion

Based on a comprehensive statistical analysis, it is concluded that lognormal significantly outperforms Weibull, which has been previously adopted as a life distribution of the commercial concentrator lattice match triple-junction GaInP/GaInAs/Ge. The best results in estimating the distribution parameters can be achieved by the censored samples from lognormal distribution compared to those of the Weibull distribution. Therefore, this work improves the estimates of the mean time to failure and warranty times of this particular family of solar photovoltaic cells. The generalization of these results to other solar cell types is under construction.

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